Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43171A

1. Three particles of mass $3m$, $2m$ and km are placed at the points whose coordinates are (1, 5), (6, 4) and (a, 1) respectively. The centre of mass of the three particles is at the point with coordinates (3, 3).

Find

(a) the value of k,

(3)

(b) the value of a.

(3)

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2. At time t seconds, where $t \geq 0$, a particle P is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]$ m s^{-2}.

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})$ m s^{-1}.

Find

(a) the velocity of P at time t seconds,

(5)

(b) the speed of P when it is moving parallel to the vector $\mathbf{i}$.

(4)

3.

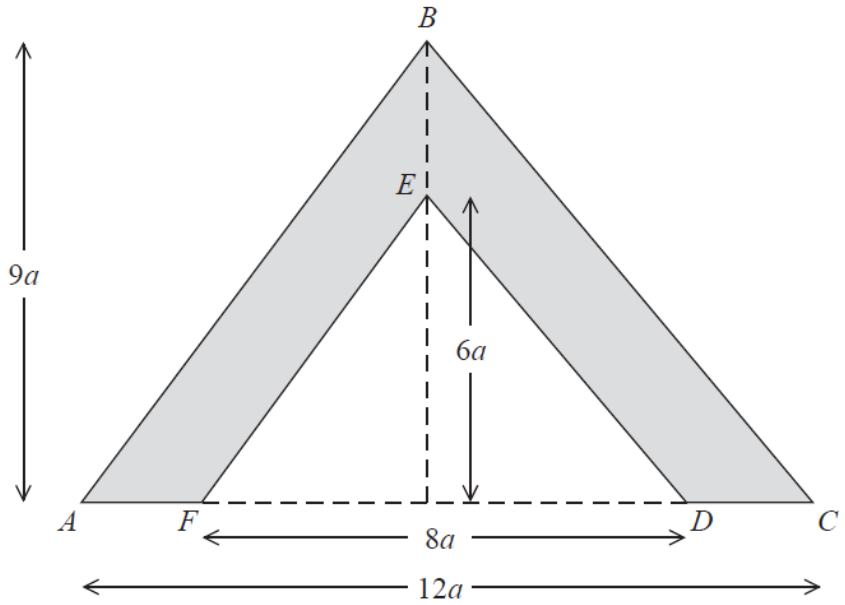

Figure 1

The uniform lamina *ABCDEF*, shown shaded in Figure 1, is symmetrical about the line through *B* and *E*. It is formed by removing the isosceles triangle *FED*, of height 6*a* and base 8*a*, from the isosceles triangle *ABC* of height 9*a* and base 12*a*.

(*a*) Find, in terms of *a*, the distance of the centre of mass of the lamina from *AC*.

(5)

The lamina is freely suspended from *A* and hangs in equilibrium.

(*b*) Find, to the nearest degree, the size of the angle between *AB* and the downward vertical.

(4)

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4. A truck of mass 1800 kg is towing a trailer of mass 800 kg up a straight road which is inclined to the horizontal at an angle α, where $\sin \alpha = \frac{1}{20}$. The truck is connected to the trailer by a light inextensible rope which is parallel to the direction of motion of the truck. The resistances to motion of the truck and the trailer from non-gravitational forces are modelled as constant forces of magnitudes 300 N and 200 N respectively. The truck is moving at constant speed v m s^{-1} and the engine of the truck is working at a rate of 40 kW.

(*a*) Find the value of *v*.

(5)

As the truck is moving up the road the rope breaks.

(*b*) Find the acceleration of the truck immediately after the rope breaks.

(4)

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5. A particle of mass m kg lies on a smooth horizontal surface. Initially the particle is at rest at a point O midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(a) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 3$ seconds.

(b) Find the value of u.

(6)

6.

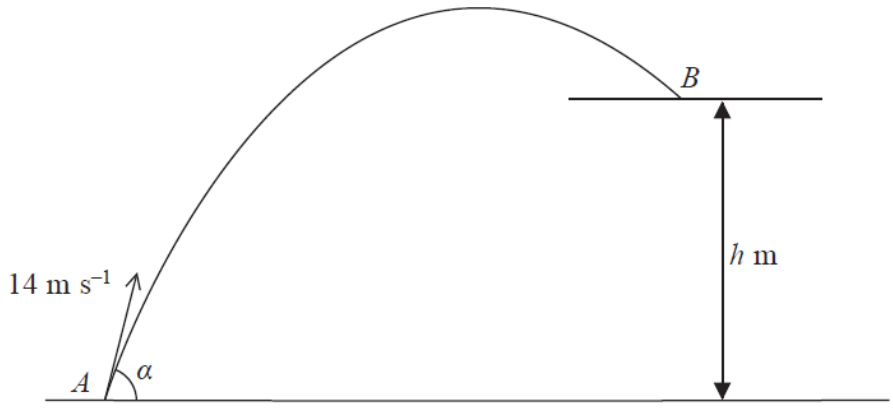

Figure 2

A small ball is projected with speed 14 m s^{-1} from a point A on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height h metres above the ground. The ball moves freely under gravity until it hits the platform at the point B, as shown in Figure 2. The speed of the ball immediately before it hits the platform at B is 10 m s^{-1}.

(a) Find the value of h.

(4)

Given that $\sin \alpha = 0.85$,

(b) find the horizontal distance from A to B.

(8)

7.

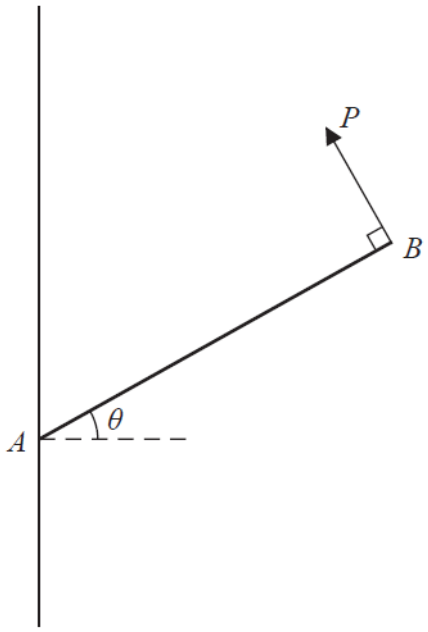

Figure 3

A uniform rod AB of weight W has its end A freely hinged to a point on a fixed vertical wall. The rod is held in equilibrium, at angle θ to the horizontal, by a force of magnitude P. The force acts perpendicular to the rod at B and in the same vertical plane as the rod, as shown in Figure 3. The rod is in a vertical plane perpendicular to the wall. The magnitude of the vertical component of the force exerted on the rod by the wall at A is Y.

(a) Show that $Y = \frac{W}{2}\left(2 - \cos^2\theta\right)$.

(6)

Given that $\theta = 45°$

(b) find the magnitude of the force exerted on the rod by the wall at A, giving your answer in terms of W.

(6)

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8. The points A and B are 10 m apart on a line of greatest slope of a fixed rough inclined plane, with A above B. The plane is inclined at 25° to the horizontal. A particle P of mass 5 kg is released from rest at A and slides down the slope. As P passes B, it is moving with speed 7 m s^{-1}.

(a) Find, using the work-energy principle, the work done against friction as P moves from A to B.

(4)

(b) Find the coefficient of friction between the particle and the plane.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks	Notes
1(a)	Moments about x axis: $3m\times5+2m\times4+km\times1=(5+k)m\times3$ $15+8+k=3k+15,$ $\qquad k=4$	M1 A1 A1 (3)	Use moments to form an equation in k. All terms required. Condone sign errors on LHS. Condone 6+k. m not required. Could be in fraction form. Correct unsimplified equation. Allow with a common factor of g cso
(b)	Moments about y axis: $3m\times1+2m\times6+km\times a=(5+k)m\times3$ $3+12+4a=27,$ $\qquad a=3$	M1 A1 A1 (3)	Use moments to form an equation in a and k (or their k) only. All terms required. m not required. Could be in fraction form. Condone 6+k. Correct unsimplified equation (follow their k if k substituted) Accept with a common factor of g. Allow after use of an incorrect value for k.
		[6]	

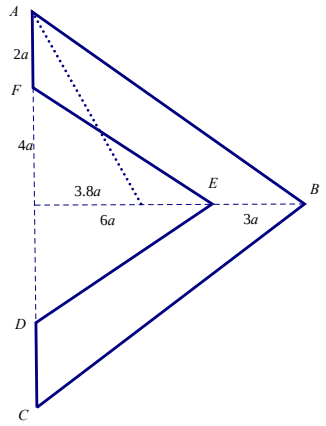
Alternative moments equations using axes through (3, 3):

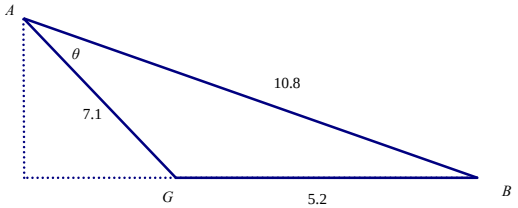
(a) Parallel to y axis $3m\times2+2m\times1=km\times2$

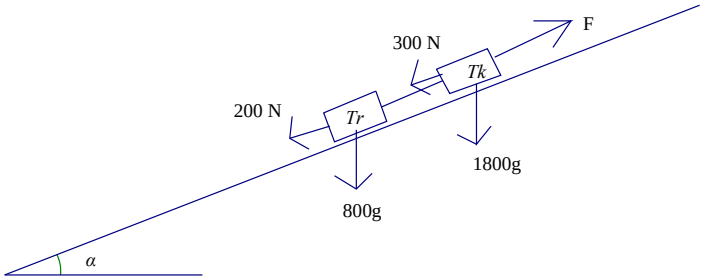
(b) Parallel to x axis $2m\times3=km(a-3)+3m\times2$

NB The vector equation on its own is not sufficient for M1 – they need to form separate equations. However, if they deduce the correct answer(s) from their vector equation full marks are available.

Question Number	Scheme	Marks	Notes
2(a)	Integrate: $\boldsymbol{v} = \left(t^3 - 2t^2\right)\mathbf{i} + \left(3t^2 - 5t\right)\mathbf{j} + \mathbf{C}$	M1	At least 3 powers going up. Condone errors in constants. Must be two separate component equations if not in vector form. Could be in column vector form. Allow with no "+ **C**"
		A2	-1 each integration error. i.e. All correct A1A1 1 error A1A0, 2 or more errors A0A0 Allow with no "+ **C**"
	$t = 3$: $\mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$ $\quad \mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$	DM1	Substitute given values to find **C**. Dependent on the previous M mark
	$\mathbf{v} = \left(t^3 - 2t^2 + 2\right)\mathbf{i} + \left(3t^2 - 5t - 2\right)\mathbf{j}$	A1 (5)	Correct velocity (any equivalent form)
(b)	Parallel to **i** $\Rightarrow 3t^2 - 5t - 2 = 0$	M1	Set **j** component of their **v** equal to zero and solve for t Correct answers imply method, but incorrect answers need to show method clearly.
	$\left(3t + 1\right)\left(t - 2\right) = 0, \quad t = 2$	A1	Correct only. Ignore $-\frac{1}{3}$ if present.
	$\left\|\mathbf{v}\right\| = 8 - 8 + 2 = 2\,(\text{m s}^{-1})$	DM1	Substitute their t to find **v**. Dependent on the previous M mark.
		A1 (4)	The **answer must be a scalar** – the Q asks for speed. Results from negative t must be rejected.
		[9]	
	A candidate who has no "+C" can score at most M1A2M0A0 M1A0M1A0		

Question Number	Scheme	Marks	Notes
3.(a)	Ratio of areas 54 : 24 : 30 (or equivalent)	B1	
	Distance of c of m from AC $3a$, $2a$	B1	
	The two B marks can be implied by a correct moments equation		All terms need to be there. Must be subtracting. Allow with g as a common factor. Allow use of an axis parallel to AC Allow in vector form.
	Moments about AC: $30d = 3a \times 54 - 2a \times 24 \left(= 114a\right)$	M1	
		A1	Correct unsimplified equation (allow in vector form)
	$d = \dfrac{114a}{30} = 3.8a$	A1 (5)	Accept any equivalent form
NB	If "a" does not appear in the solution at all, mark the work as a misread. If "a" appears and disappears then mark as given in the scheme.		

Question Number	Scheme	Marks	Notes
3(b)	Correct triangle and use of $\tan^{-1}$ or equivalent	M1	Find a relevant angle using their d. Condone ratio the wrong way up. Allow $\tan^{-1}\left(\frac{\text{their } d}{\text{their } \bar{x}}\right)$
alt1	$\tan^{-1}\left(\frac{3.8a}{6a}\right)$, $\tan^{-1}\left(\frac{6a}{3.8a}\right)$, 32.3… or 57.65….	A1	
	Required angle = $\tan^{-1}\left(\frac{9a}{6a}\right)-\tan^{-1}\left(\frac{3.8a}{6a}\right)=23.96.....=24°$	DM1 A1 (4)	Correct method for the required angle. Dependent on the previous M mark. Only. The Q asks for answers to the nearest degree.
alt2		M1	Identify the correct triangle and find the lengths of the sides
		A1	All correct (accept lengths as unsimplified calculations using Pythagoras)
	$\cos\theta=\frac{10.8^2+7.1^2-5.2^2}{2\times10.8\times7.1}$	DM1	Use trigonometry to find θ
	$\theta=24°$	A1	
		[9]	

Question Number	Scheme	Marks	Notes
4			
(a)	Constant speed $F = 200 + 800g\sin\alpha + 300 + 1800g\sin\alpha \left(= 1774\right)$	M1 A2	Complete method to an equation in "F". Requires all the terms, including resolution of the weights. Condone sign errors and sin/cos confusion. g missing from both weights is a single error. Penalise trig once only. -1 each error. i.e. A1A1 if no errors A1A0 one error seen, A0A0 two or more errors
	$40000 = Fv \left(= 1774v\right)$ $v = 22.5$	M1 A1 (5)	**Use of** $P = Fv$. Allow with F or their F. Independent of the first M1 Accept 23. (maximum 3sf following use of 9.8)
(b)	"1774" $- 300 - 1800g\sin\alpha \left(= 592\right) = 1800a$ $F = ma$: $a = \dfrac{F}{1800} = 0.32888... = 0.33 \left(\text{m s}^{-2}\right)$	M1 A2 A1 (4)	New equation of motion for the truck. Follow their 1774. Requires all the terms, including resolution of the weights. Condone sign errors and sin/cos confusion. Allow with their 1774. -1 each error. i.e. A1A1 no errors, A1A0 one error, A0A0 two or more errors Accept 0.329
		[9]	

Question Number	Scheme	Marks	Notes
5(a)	Speed after impact = $\frac{2}{3}u$	B1	Allow for velocity $= -\frac{2u}{3}$
	Impulse = change in momentum = $\pm\left(m.\text{their}\frac{2}{3}u - m.(-u)\right)\left(=\frac{5}{3}mu\right)$	M1	Need to consider momentum before and after the collision and use change of direction.
	$\lambda = \frac{5}{3}$	A1 (3)	cso
(b)	Speed after second collision = $e^2u = \frac{4}{9}u$	B1	Allow negative
	Total time taken = $\frac{1}{u} + \frac{2}{eu} + \frac{1}{e^2u}\left(=\frac{1}{u}+\frac{3}{u}+\frac{9}{4u}\right)$	M1	Use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ to find the total time in terms of u. (At least one term dealt with correctly)
		A2	-1 each error
Alt for M1A2	Ratio of times for the three sections is $\frac{1}{3}:1:\frac{3}{4}$	M1 A2	Or equivalent. -1 each error
	$\frac{4}{u}+\frac{9}{4u}=\frac{25}{4u}=3,\quad u=\frac{25}{12}$ o.e.	DM1 A1 (6)	Use total time = 3 and solve for u Accept 2.08 and 2.1 (or better)
		[9]	
	SC The candidate who only considers the first return to O can score the first M1 in (b) for $\frac{1}{u}+\frac{1}{eu}$ giving 1/6 marks		

Question Number	Scheme	Marks	Notes
6(a)	Considering energy: $\frac{1}{2}m\times14^2=\frac{1}{2}m\times10^2+mgh$ $h=\frac{48}{g}=4.90$	M1 A2 A1 (4)	All terms required. Terms need to be of the correct form but condone sign errors. -1 each error in the unsimplified equation Accept $\frac{48}{g}$. Maximum 3 s.f. if they go in to decimals.
alt(a)	Initial $v_y=14\sin\alpha$ Final $v_y=\sqrt{100-14^2\cos^2\alpha}$ $100-196\cos^2\alpha=196\sin^2\alpha-2gh$ $h=\frac{48}{g}=4.90$	M1A2 A1 (4)	Using $v^2=u^2+2as$ on the vertical components of speed. -1 each error in the unsimplified equation Accept in exact form. Maximum 3 s.f. if they go in to decimals.
NB	Using $v^2=u^2+2as$ with 10 and 14 is M0		
NB	In part (a) they must be solving the general case, not using 0.85. However, the marks in (b) are all available if they solve the specific case in (a).		
(b)	Vertical distance: $h=14\sin\alpha t-\frac{1}{2}\times9.8t^2$ $4.9t^2-11.9t+h=0$ $t=\frac{11.9\pm\sqrt{11.9^2-4\times4.9^2}}{9.8}$ $t=1.903.....$ Horizontal distance: $x=14\cos\alpha\times t$ =14.0 (m)	M1 A2 DM1 A1 M1 A1 A1 (8)	A complete method to find an equation in *t.* Must involve trig condone sin/cos confusion Correct in *h* or their *h*. -1 each error Solve a 3 term quadratic for *t*. Needs their value for *h* now. 1.9 or better Method for the horizontal distance. Condone consistent sin/cos confusion Correct for their positive *t* Accept 14

Question Number	Scheme	Marks	Notes
Alt (b)	Vertical speed = $\sqrt{100-(14\cos\alpha)^2}$ (=6.75) $v=u+at=14\times0.85-9.8t \quad (-6.75=11.9-9.8t)$ $t=1.903.....$ Horizontal distance: $x=14\cos\alpha\times t$ =14.0 (m)	M1 A2 DM1 A1 M1 A1 A1 (8)	A complete method to find the vertical component of the speed at *B*. Correct insimplified. -1 each error. Use their vertical component to find *t* 1.9 or better Method for the horizontal distance. Correct for their positive *t* Accept 14
		[12]	
NB	Candidates with a false method leading to 4.9 in (a) score at most M1A1A1DM1A0M1A1A0 if they use their result in (b). This error does not affect the alt (b) approach		

Question Number	Scheme	Marks	Notes
7(a)	Resolving vertically: $Y + P\cos\theta = W$	M1 A1	Needs all 3 terms. Condone sign errors and sin/cos confusion. Condone Wg
	Moments about A: $Wl\cos\theta = 2lP$	M1 A1	Terms need to be of the correct structure, but condone l implied if not seen.
	$P = \frac{W\cos\theta}{2} \Rightarrow Y = W - \frac{W\cos^2\theta}{2} = \frac{W}{2}\left(2 - \cos^2\theta\right)$ ****	DM1 A1 (6)	Substitute for P to obtain simplified Y Requires both preceding M marks Obtain **given result** correctly.
	NB $W + Y = P\cos\theta$ with correct conclusion is possible		
	They need to find two independent equations that do not include X. If they have equations involving X they need to attempt to eliminate X before they score any marks		
(b)	$\theta = 45° \Rightarrow Y = \frac{3W}{4}$	B1	
	$X = P\sin 45$	M1	Resolving horizontally. Accept in terms of θ.
	$= \frac{W\cos 45}{2}.\sin 45\left(= \frac{W}{4}\right)$	DM1 A1	Express X in terms of W. Accept in terms of θ. Requires preceding M mark. Correct unsimplified but substituted.
	Resultant at A = $\frac{W}{4}\sqrt{3^2 + 1^2} = \frac{W\sqrt{10}}{4}$ $(0.79W)$	DM1 A1 (6)	Use of Pythagoras with X, Y in terms of W only. Dependent on the first M1 Or equivalent $(0.79W$ or better)
	Alternative moments equations: about the centre $Pl + X\sin\theta l = y\cos\theta l$ About the point where the lines of action of P and X intersect $Y \times \frac{2l}{\cos\theta} = W\left(\frac{2l}{\cos\theta} - l\cos\theta\right)$		

Question Number	Scheme	Marks	Notes
8(a)	Work done against friction = Loss in GPE– Gain in KE $= 5\times 9.8\times 10\sin 25 - \frac{1}{2}\times 5\times 7^2 = 84.58...$ = 85 (J) (84.6)	M1 A2 A1 (4)	Must be using Work-energy principle Needs to consider (work done) KE & GPE and no other terms. Condone sign errors. Watch out for incorrect solutions including both change in GPE and the work done against the weight – this is a method error. -1 each error Max 3 s.f. Must be +ve. Accept as $10F = 84.6$ or equiv.
(b)	$F = \mu R = \mu \times 5g\cos 25$ Work done = $10F = 10\mu \times 5g\cos 25$ = their85 $\mu = 0.19$	M1 A1 M1 A1ft A1 (5)	Resolve to find $F_{\text{max.}}$ g missing is an accuracy error Correct unsimplified Use of work done = force x distance to form an equation for μ Correct unsimplified equation for their $10F$ Accept 0.190
		[9]	
altb	$F = \mu R = \mu \times 5g\cos 25$	M1 A1	Resolve to find $F_{\text{max.}}$ g missing is an accuracy error Correct unsimplified
	$v^2 = u^2 + 2as \rightarrow 49 = 20a \rightarrow a = \frac{49}{20}$ N2L $\rightarrow 5\times\frac{49}{20} = 5g\sin 25 - \mu\times 5g\cos 25$	M1 A1	**Complete** method to an equation in μ Correct unsimplified equation
	$\mu = 0.19$	A1 (5)	Accept 0.190